UNIVERSITY OF THE PUNJAB

B.A. / B.Sc. Part – II

Annual Examination – 2019

Roll No. ....

Subject: Mathematics
PAPER: Optional

TIME ALLOWED: 3 Hrs.
MAX. MARKS: 100

Note: Attempt any *FIVE* questions in all, selecting at least *TWO* questions from each section.

SECTION-I

Q.1	(a)	Let $z = acr\ sin\left(\frac{x}{y}\right)$. Verify that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$	(10)
	(b)	Evaluate $\lim_{x\to 0}(1+\sin x)^{\cot x}$	(10)
Q.2	(a)	Find $\frac{dy}{dx}$ when $x = \frac{a(1-t^2)}{1+t^2}$, $y = \frac{2bt}{1+t^2}$	(10)
	(b)	Evaluate $\lim_{x\to 0}(\tan x)^{\sin 2x}$	(10)
Q.3	(a)	Find the Maclaurin series of $f(x) = \cos x$	(10)
	(b)	If $z = f(x,y) = e^{-x}\cos y$ then show that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$	(10)
Q.4	(a)	Solve $\int \frac{dx}{x\sqrt{a^2+x^2}}$	(10)
	(b)	Solve $\int e^x\left(\frac{1+x\ln x}{x}\right)dx$	(10)
Q.5	(a)	Solve the differential equation $(\sin x+\cos x)dy+(\cos x-\sin x)dy=0$.	(10)
	(b)	Find the extreme values of the function $f(x)=\sin x\cos 2x$	(10)

P.T.O.

SECTION-II

Q.6	(a)	Show that the set of vectors $\{(1,2,3),(0,1,2),(0,0,1)\}$ generates R^3.	(10)
	(b)	Determine whether the vectors are linearly independent or not? $v_1=(1,1,1), v_2=(0,1,1), v_3=(0,1,-1)$.	(10)
Q.7	(a)	Show that the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$ is nilpotent	(10)
	(b)	If possible, find the inverse of the matrix $\begin{bmatrix} 1 & 2 & -3 \\ 1 & -2 & 1 \\ 5 & -2 & -3 \end{bmatrix}$	(10)
Q.8	(a)	Solve the system of linear equations $2x+z=1,\quad 2x+4y-z=-2,\quad x-8y-3z=2$	(10)
	(b)	Find the reduced echelon form of the matrix $\begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{bmatrix}$	(10)
Q.9	(a)	Show that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$.	(10)
	(b)	Prove that $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$	(10)
Q.10	(a)	Show that $\begin{vmatrix} a & 1 & 1 & 1 \\ 1 & a & 1 & 1 \\ 1 & 1 & a & 1 \\ 1 & 1 & 1 & a \end{vmatrix} = (a-1)^3(a+3)$	(10)
	(b)	Distinguish between basis and dimension of a vector space. Distinguish between linear independence and dependence of vectors.	(10)

UNIVERSITY OF THE PUNJAB

B.A. / B.Sc. Part – II

Annual Examination – 2019

Roll No. ....

Subject: Mathematic General-II
PAPER: (Mathematical Methods (Geomt, Series, Compl. No. LA, DE)

TIME ALLOWED: 3 Hrs.
MAX. MARKS: 100

Note: *Attempt SIX questions in all by selecting TWO questions from Section – I. TWO questions from Section – II. One question from Section – III and ONE Question from Section – IV.*

SECTION-I

Q.1 (a) Find the squares of all the 5^{th} roots of $\frac{1}{2}+\frac{\sqrt{3}}{2}i$. 9

(b) If $\tan(\alpha + i\beta) = x + iy$, show that $x^2 + y^2 - 2y \coth 2\beta = -1$. 8

Q.2 (a) Show that $2 + i = \sqrt{5}\, e^{i\tan^{-1}\left(\frac{1}{2}\right)}$. 9

(b) Find the sum of $\sinh\theta + \frac{\sinh 2\theta}{2!} + \frac{\sinh 3\theta}{3!} + \ldots\ldots$ 8

Q.3 (a) Test the series $\sum_{1}^{\infty} \frac{e^{\arctan n}}{1+n^2}$ convergence or divergence. 9

(b) Use appropriate test to determine the convergence or divergence of $\sum_{2}^{\infty} \frac{(2n+1)(3^n+1)}{4^n+1}$. 8

Q.4 (a) Test the absolute convergence of the series $\sum_{1}^{\infty} \frac{(-1)^n (n+2)}{n(n+1)}$. 9

(b) Find the radius of convergence and interval of convergence of $\sum_{n=1}^{\infty}\left(1+\frac{1}{2}+\frac{1}{3}+\ldots\frac{1}{n}\right)x^n$. 8

SECTION-II

Q.5 (a) Prove that in any (right angled) triangle, the median to the hypotenuse is equal to one half of the hypotenuse. 9

(b) Prove that $[\bar{a}+\bar{b}\,,\ \bar{b}+\bar{c}\,,\ \bar{c}+\bar{a}] = 2\,[\bar{a}\ \bar{b}\ \bar{c}]$. θ

Q.6 (a) Show that the line joining the points A(2, –3, –1) and B(8, –1, 2) has equations $\frac{1}{6}(x-2) = \frac{1}{2}(y+3) = \frac{1}{2}(z+2)$.

(b) Find directional derivative of $\phi = e^{2x-y+z}$ at P(1, 1, 1) in the direction of $-3i + 5j + 6k$. 8

Q.7 (a) Transform the equations of the planes $3x - 4y + 5z = 0$ and $2x - y - 2z = 5$ to normal forms and find measure of the angle between them. 9

(b) Find equation of the plane through the straight line $x + y - z = 0 = 2x - y + 3z - 5$ and perpendicular to the coordinate planes. 8

Q.8 (a) Find equation of the cone whose directrix is $4x^2 + (y-2)^2 = 4$, $z = 3$ and vertex A = (0, 0, 0). 9

(b) Find the direction of Qibla at Quetta, latitude = $24^{\circ}\ 51.5'$ N and longitude $67^{\circ}\ 0'$E. 8

SECTION-III

Q.9 (a) Let A and B are distinct $n \times n$ matries with real entries. If $AB^2 = BA^2$ and $A^3 = B^3$, show that $A^2 + B^2$ is not invertible. 8

(b) Show that the vectors (1, –2, 4, 1), (2, 1, 0, –3) and (1, –6, 1, 4) are linearly independent. 8

Q.10 (a) Find rank of the matrix $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & 0 \\ -2 & -1 & 3 \\ -1 & 4 & -2 \end{bmatrix}$. 8

(b) Find Eigen values of $\begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$. 8

SECTION-IV

Q.11 (a) Solve the differential equation $y(2xy + e^x)\,dx - e^x\,dy = 0$. 8

(b) Find equation of orthogonal trajectories for the curve $r^n = a^n \cos n\theta$. 8

Q.12 (a) Solve $y'' - 4y = 2 - 8x$, $y(0) = 0$, $y'(0) = 5$. 8

(b) Solve $(x^2D^2 + 2xD - 6)\,y = 10x^2$; $y(1) = 1$, $y'(1) = -6$. 8

UNIVERSITY OF THE PUNJAB

B.A. / B.Sc. Part – II

Annual Examination – 2019

Roll No. ....

Subject: Mathematics A Course-II
PAPER: (Linear Algebra and Differential Equations)

TIME ALLOWED: 3 Hrs.
MAX. MARKS: 100

Note: ***Attempt SIX Questions by selecting TWO questions from Section-I, ONE Question from Section-II &Section-III and TWO questions from Section-IV.***

SECTION –I

Q.1. (a) Let A and B be idempotent matrices i.e. $A^2=A$, $B^2=B$ show that

i. If AB=BA then AB is idempotent.

ii. If A^T is idempotent so is A. Is the sum of two Idempotent matrices idempotent?

(b). Prove that for an invertible matrix A, det A≠0 and det $(A^{-1})= \frac{1}{\det A}$ (9,8)

Q.2. (a) Find the solution of the system of linear equations by Gauss Jordan Elimination method

$$2x_1 - x_2 - x_3 = 4, 3x_1 + 4x_2 - 2x_3 = 11, 3x_1 - 2x_2 + 4x_3 = 11$$

(b) Find, by Adjoint method, the inverse of $A=\begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 7 \end{bmatrix}$ (9,8)

Q.3. (a) Show that the yz-plane w={(0,y,z):y, z ε R} is spanned by (0,1,2),(0,2,3) and (0,3,1).

(b) Let V be the vector space of all functions defined on R to R. Check whether the vectors 2, $4\sin^2 x$, $\cos^2 x$ are linearly indepent in V. (9,8)

Q 4 (a) Find the rank of matrix $A=\begin{bmatrix} 2 & 1 & 3 & 5 \\ 15 & 8 & 1 & 12 \\ 11 & 5 & 8 & 6 \\ 12 & 8 & 7 & 10 \end{bmatrix}$ and write an echelon matrix row equivalent to A.

(b) Find the matrix of linear transformation T: $R^3 \to R^4$ defined by $T(x_1, x_2, x_3) = (x_1 + x_2, x_2 + x_3, x_1 - x_3, x_1)$ with respect to the standard basis for R^3 and R^4 (9,8)

P.T.O.

UNIVERSITY OF THE PUNJAB

B.A. / B.Sc. Part – II

Annual Examination – 2019

Roll No. ....

Subject: Mathematic General-II

PAPER: (Mathematical Methods (Geomt, Series, Compl. No. LA, DE)

TIME ALLOWED: 3 Hrs.

MAX. MARKS: 100

Note: Attempt SIX questions in all by selecting TWO questions from Section – I. TWO questions from Section – II. One question from Section – III and ONE Question from Section – IV.

SECTION-I

Q.1 (a) Express $\sin^6\theta$ in the series of sines or cosines of multiple of θ if $x = \cos\theta + i\sin\theta$. 9

(b) If $\sin(\theta + i\phi) = \cos\alpha + i\sin\alpha$, prove that $\cos^2\theta = \pm\sin\alpha$. 8

Q.2 (a) If $\log\sin(x + iy) = u + iv$, show that $e^{2y} = \dfrac{\cos(x - v)}{\cos(x + v)}$. 9

(b) Find the sum of the infinite series $\dfrac{c^2}{2!}\sin 2\theta - \dfrac{c^4}{4!}\sin 4\theta + \dfrac{c^6}{6!}\sin 6\theta + \ldots\ldots$ 8

Q.3 (a) Test for the convergence or divergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(n+1)[\ln(n+1)]^2}$ 9

(b) Determine convergence or divergence of the series. $\dfrac{2}{5} + \dfrac{2.4}{5.8} + \dfrac{2.4.6}{5.8.11} + \dfrac{2.4.6.8}{5.8.11.14} + \ldots$ 8

Q.4 (a) Test the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n!}{(2n)!}$ for (i) absolute convergence (ii) conditional convergence (iii) divergence 9

(b) Find the radius of convergence and the interval of convergence for the series $\displaystyle\sum_{n=0}^{\infty} \frac{2^n x^n}{\ln(x+2)}$. 8

SECTION-II

Q.5 (a) Prove that in any right triangle Δ, the median to the hypotenuse is equal to one half of the hypotenuse. 9

(b) Prove that $\bar{A} = 5\bar{a} + 6\bar{b} + 7\bar{c}$, $\bar{B} = 7\bar{a} - 8\bar{b} + 9\bar{c}$ and $\bar{C} = 3\bar{a} + 20\bar{b} + 5\bar{c}$ are coplanar. 8

P.T.O.

UNIVERSITY OF THE PUNJAB

B.A. / B.Sc. Part – II

Annual Examination – 2019

Roll No. ....

Subject: Mathematics A Course-II
PAPER: (Linear Algebra and Differential Equations)

TIME ALLOWED: 3 Hrs.
MAX. MARKS: 100

***Note:** Attempt SIX Questions by selecting TWO questions from Section-I, ONE Question from Section-II &Section-III and TWO questions from Section-IV.*

Section - I

Q.1. (a) If A and B are symmetric matrices, then prove that AB is symmetric if and only if A and B commute. (9, 8)

(b) For value of α is the matrix $A=\begin{bmatrix} -\alpha & \alpha-1 & \alpha+1 \\ 1 & 2 & 3 \\ 2-\alpha & \alpha+3 & \alpha+7 \end{bmatrix}$ singular?

Q.2. (a) Solve the system of equationsby Gauss-Jordan elimination method:

$$2x_1 - x_2 + 3x_3 = 3$$
$$3x_1 + x_2 - 5x_3 = 0$$
$$4x_1 - x_2 + x_3 = 3$$

(9, 8)

(b) Show that $\begin{vmatrix} 1 & 1 & 1 \\ \alpha & \beta & \gamma \\ \alpha^3 & \beta^3 & \gamma^3 \end{vmatrix} = (\beta-\gamma)(\gamma-\alpha)(\alpha-\beta)(\alpha+\beta+\gamma)$

Q.3. (a) Determine whether the set $S=\{(1,1,2),(1,0,1),(2,1,3)\}$ spans R^3. (9, 8)

(b) Let V be the real vector space of all function defined on R into R. Determine whether the vectors $\sin^2 x$, $\cos^2 x$, $\cos 2x$ are linearly dependent in V.

Q.4. (a) A linear transformation $T: U \to V$ is one-to-one if and only if N(T)={0}.

(b) Find the matrix of the linear transformation $T: R^3 \to R^4$ defined by $T(x_1, x_2, x_3) = (x_1 + x_2, x_2 + x_3, x_1 - x_3, x_1)$ with respect to the standard bases of R^3 and R^4. (9, 8)

Section - II

Q.5. (a) Find a unit vector orthogonal to both (1, 1, 2) and (0, 1, 3) in R^3.

(b) Find an orthogonal matrix whose first row is a multiple of (1, 1, 1). (8, 8)

Q.6. (a) Prove that the eigen values of symmetric matrix are all real.

(b) For symmetric matrix $A=\begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix}$ find an orthogonal matrix P for which P^TAP is diagonal. (8, 8)

P.T.O.

UNIVERSITY OF THE PUNJAB

B.A. / B.Sc. Part – II

Annual Examination – 2019

Roll No. ....

Subject: Mathematics B Course-II
PAPER: (Mathematical Methods, Group Theory & Matrix Space)

TIME ALLOWED: 3 Hrs.
MAX. MARKS: 100

Note: ***Attempt SIX quuestion in all by selecting TWO questions from Section-I & Section-II, ONE question from Section-III & Section-IV.***

SECTION I

Q1(a) Express $cos^4\,\theta$ in the series of cosines of multiples of θ. (9)

(b)Find the sum of Infinite series $\sin\theta - \frac{1}{2}\sin 3\theta + \frac{1}{3}\sin 5\theta$ .... (8)

Q2(a) If Z is a complex Number , then show that $\sin^{-1}h\,Z = \log(Z + \sqrt{Z^2+1})$. (9)

(b) Prove that all Normal lines of the sphere $x^2 + y^2 + z^2 = a^2$ pass through the center of the sphere. (8)

Q3(a) Find the maximum value of $f\,(x,y,z) = x^4 + y^4 + z^4$ subject to x+y+z= 1. (9)

(b) Verify that $f_{xy} = f_{yx}$ when $f(x,y) = \ln(e^x + e^y)$ (8)

Q4(a) Find the mass of a sphere of radius r if the density varies Inversely as the square of the distance from the centre. (9)

(b) Evaluate $\int_2^4 \int_y^{8-y} y\,dx\,dy$. (8)

SECTION II

Q5(a) Test the series $\sum_1^{\infty} \frac{2^n}{n(n+2)}$. (9)

(b)Whether the alternating series is converges or diverges $\sum_1^{\infty}(-1)^{n-1}\frac{n+4}{n^2+n}$. (8)

Q6(a) If a=bq+r then show that (a,b) = (b,r). (9)

(b) Prove that $64 \,|\, 7^{2n} + 16n - 1$. (8)

Q7(a) Solve the converges $11\,x^9 + 1 \equiv 0 \pmod{29}$. (9)

(b) Investigate the Behavior of Eulers series

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} \;\dots\dots + \frac{1}{n^2}\dots\dots$$ (8)

P.T.O.

Q8(a) Test the series $\sum_1^\infty \frac{2.4.6\ldots\ldots(2n)}{1.3.5\ldots\ldots(2n-1)}$ (9)

(b) Define Fermats number and Prove that they are Co- Prime. (8)

SECTION III

Q9(a) Every subgroup of a cyclic group is cyclic. (8)

(b) If $\alpha = \begin{pmatrix} 1\,2\,3\,4\,5\,6\,7 \\ 3\,5\,4\,1\,7\,2\,6 \end{pmatrix}$ find the Inverse of α (8)

Q10(a) If H is a subgroup of a Group G. Then show that $H.H = \{h_1h_2 : h_1, h_2 \in H\} = H$. (8)

(b)Let G be a cyclic group of order n generated by a.Then For each positive divisor d of n,

there is a unique subgroup of G of order d. (8)

SECTION IV

Q11(a)Show that an open sphere in R is just the open Interval. (8)

(b) Let x be a limit point of a sunset A of a metric space X, Then every nbhd of x contains

infinitely many points of A. (8)

Q12(a) Let A and B be any two subset of a Topological space. Then show that

$Int\ (AUB) \supseteq Int\ (A) U\ Int\ (B)$ (8)

(b) A subset of a metric space is closed if and only if It contains its Boundary. (8)

UNIVERSITY OF THE PUNJAB

B.A. / B.Sc. Part – II

Annual Examination – 2019

Roll No. ....

Subject: Mathematics B Course-II
PAPER: (Mathematical Methods, Group Theory & Matrix Space)

TIME ALLOWED: 3 Hrs.
MAX. MARKS: 100

Note: Attempt SIX quuestion in all by selecting TWO questions from Section-I & Section-II, ONE question from Section-III & Section-IV.

SECTION I

Q1(a) Express $cos^4\,\theta$ in the series of cosines of multiples of θ. (9)

(b)Find the sum of Infinite series $\sin\theta - \frac{1}{2}\sin 3\theta + \frac{1}{3}\sin 5\theta$ .... (8)

Q2(a) If Z is a complex Number , then show that $\sin^{-1}h\,Z = \log(Z + \sqrt{Z^2+1})$. (9)

(b) Prove that all Normal lines of the sphere $x^2+y^2+z^2=a^2$ pass through the center of the sphere. (8)

Q3(a) Find the maximum value of $f\,(x,y,z) = x^4+y^4+z^4$ subject to x+y+z= 1. (9)

(b) Verify that $f_{xy} = f_{yx}$ when $f(x,y) = \ln(e^x + e^y)$ (8)

Q4(a) Find the mass of a sphere of radius r if the density varies Inversely as the square of the distance from the centre. (9)

(b) Evaluate $\int_2^4 \int_y^{8-y} y\,dx\,dy$. (8)

SECTION II

Q5(a) Test the series $\sum_1^\infty \frac{2^n}{n(n+2)}$. (9)

(b)Whether the alternating series is converges or diverges $\sum_1^\infty (-1)^{n-1}\ \frac{n+4}{n^2+n}$. (8)

Q6(a) If a=bq+r then show that (a,b) = (b,r). (9)

(b) Prove that 64 .| $7^{2n}+16n-1$. (8)

Q7(a) Solve the converges $11\,x^9+1 \equiv 0\ (mod\ 29)$. (9)

(b) Investigate the Behavior of Eulers series

$$\sum_{n=1}^\infty \frac{1}{n^2} = \frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}\ \ldots\ldots + \frac{1}{n^2}\ldots\ldots$$ (8)

P.T.O.

Q8(a) Test the series $\sum_1^{\infty} \frac{2.4.6.........(2n)}{1.3.5..........(2n-1)}$ (9)

(b) Define Fermats number and Prove that they are Co- Prime. (8)

SECTION III

Q9(a) Every subgroup of a cyclic group is cyclic. (8)

(b) If $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 5 & 4 & 1 & 7 & 2 & 6 \end{pmatrix}$ find the Inverse of α (8)

Q10(a) If H is a subgroup of a Group G. Then show that $H.H = \{h_1h_2 : h_1, h_2 \in H\} = H$. (8)

(b)Let G be a cyclic group of order n generated by a.Then For each positive divisor d of n,

there is a unique subgroup of G of order d. (8)

SECTION IV

Q11(a)Show that an open sphere in R is just the open Interval. (8)

(b) Let x be a limit point of a sunset A of a metric space X, Then every nbhd of x contains

infinitely many points of A. (8)

Q12(a) Let A and B be any two subset of a Topological space. Then show that

$Int\ (A \cup B) \supseteq Int\ (A) \cup Int\ (B)$ (8)

(b) A subset of a metric space is closed if and only if It contains its Boundary. (8)

UNIVERSITY OF THE PUNJAB

B.A. / B.Sc. Part – II

Annual Examination – 2019

Roll No. ....

Subject: Mathematics B Course-II
PAPER: (Mathematical Methods, Group Theory & Matrix Space)

TIME ALLOWED: 3 Hrs.
MAX. MARKS: 100

Note: ***Attempt SIX quuestion in all by selecting TWO questions from Section-I & Section-II, ONE question from Section-III & Section-IV.***

SECTION I

Q.1 (a) Separate into real and imaginary part$(\alpha + i\beta)^{(P+iq)}$. (9)

(b)Find the sum of Infinite series $\sin\theta - \frac{1}{2}\sin 3\theta + \frac{1}{3}\sin 5\theta$ ……… (8)

Q2(a) If Z is a complex Number, then show that $\sin^{-1} h\, Z = \log(Z + \sqrt{Z^2+1})$. (9)

(b) Prove that all Normal lines of the sphere $x^2 + y^2 + z^2 = a^2$ pass through the center of thesphere. (8)

Q3(a) Find the maximum value of $f(x,y,z) = x^4 + y^4 + z^4$ subject to x+y+z= 1.

(b) State and prove Eulers theorem. (9, 8)

Q.4 (a) Find the area bounded by the parabola $y = x^2$and the straight line $y = 2x + 3$ (9,

(b) Evaluate $\int_2^4 \int_y^{8-y} y\, dx\, dy$. (8)

SECTION II

Q5 (a) Test the series $\sum_1^\infty \frac{2}{\sqrt{n+1}}$

(b) Test the series $\sum_1^\infty n\left(\frac{\pi}{n}\right)^n$ (9, 8)

Q6(a) If a=bq+r then show that (a,b) = (b,r).(9)

(b) Prove that $64 \mid 7^{2n} + 16n - 1$. (8)

Q7(a) Solve the converges $11x^9 + 1 \equiv 0 \ (mod\ 29)$. (9)

(b) Investigate the Behavior of Eulers series

$\sum_{n=1}^\infty \frac{1}{n^2} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2}$ …….+ $\frac{1}{n^2}$……… (8)

P.T.O.

Q8(a) Using Integral Test show that Harmonic series $\sum_1^\infty \frac{1}{n}$ is divergent. 9,8

(b) Define a Prime Divisor and prove that every Integer n>1 has a prime divisor.

SECTION III

Q9(a)Let G be a group and H is a subgroup of G.Then the set $aHa^{-1} = \{aha^{-1}: h \in H\}$ is a subgroup of G.

(b) If $\alpha = \binom{1\,2\,3\,4\,5\,6\,7}{3\,5\,4\,1\,7\,2\,6}$find the Inverse of α 8,8

Q10(a) If H is a subgroup of a Group G. Then show that $H.H = \{h_1h_2 : h_1, h_2 \in H\} = H.$ (8)

(b)Let G be a cyclic group of order n generated by a.Then For each positive divisor d ofn,

there is a unique subgroup of G of order d. (8)

SECTION IV

Q11(a)Show that an open sphere in R is just the open Interval. (8)

(b) Let x be a limit point of a sunset A of a metric space X, Then every nbhdof x contains

infinitely many points of A. (8)

Q12(a) If A and B are two subsets of a metric space X.Then A⊆ B Implies that $A^a \subseteq B^a$. 8,8

(b) If A and B are two subsets of a metric space X. Then $\overline{AUB} = \bar{A}U\bar{B}$